

# Comparative Study of the detection algorithms in MIMO

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**Abstract-** Wireless communication systems are gaining importance especially with respect to mobiles phones and data devices because of their ease of use and mobility. The need for high data rate is growing since the multimedia applications are gaining popularity which needs high data rate and quality of service. Multiple Input Multiple Output (MIMO) transmission system is one of the recent and the most promising approach of the Smart Antenna Technology which uses multiple antennas in the transmitter and the receiver side and is currently followed for high-rate wireless communication. The capacity of MIMO systems is much better when compared to all other antenna configurations like SISO, SIMO, MISO. In MIMO, many receiver algorithms have been used for the detection of the transmitted symbols. This paper discusses some of the algorithms used and they are compared based on complexity and BER performance. Out of the discussed algorithms, Maximum Likelihood (ML) is found to be the best in terms of BER but the complexity increases exponentially with increase in number of transmitters. The Sphere decoding algorithm is gradually replacing ML as it reduces the computational complexity while maintaining the same performance as that of ML. The algorithms are simulated in MATLAB and BER performances are validated.

**Keywords—** Multiple Input Multiple Output (MIMO) systems, Single Input Single Output (SISO), Single Input Multiple Output (SIMO), Multiple Input Single Output (MISO), Maximum Likelihood detection (ML), and Sphere decoding algorithm.

## I. INTRODUCTION

With the integration of Internet and multimedia application in next generation wireless communication, the demand for wide-band high data rate communication services is growing. As the available radio spectrum is limited, high data rates can be achieved only by designing more efficient signalling techniques. Recent trends in information technology have shown that large gain in the capacity of communication over wireless channels is feasible in Multiple Input Multiple Output (MIMO) systems. The MIMO channel can be constructed with multiple antenna arrays on either side of wireless link.

Multiple Input Multiple Output (MIMO) has been one of the most promising technologies to improve performance of a wireless link. The advantage of MIMO system is to benefit the users with multipath propagation. MIMO improves the capacity and combats fading by different diversity techniques. Wireless systems using MIMO can significantly

improve the spectral efficiency of a system. The two main techniques used in MIMO are diversity technique and the spatial multiplexing technique. The former improves the signal-to-noise ratio (SNR) and improves reliability while the latter improves the channel capacity without additional bandwidth. Spatial multiplexing is the method which is in practice nowadays.

The challenge in the practical realisation of a MIMO system lies in the efficient implementation of a detector in separating the spatially multiplexed signals. While coded MIMO schemes offer better performance than the uncoded and modulated schemes, its hardware complexity is practically formidable, especially for a system with more than 4 antennas on both transmitter and receiver. We start with uncoded MIMO schemes and also do a comparative study on the different receiver algorithms from both performance and complexity point of view.

This paper is organised as follows. Section II discusses the basic model of a MIMO system. Section III deals with the various algorithms used in the receiver side. These algorithms are simulated and their performance is compared in Section IV. Section V concludes the paper by identifying the Sphere decoder replacing the ML algorithm as its less complex.

## II. MIMO SYSTEM MODEL

We consider MIMO systems with  $M_T$  transmit antennas in the transmitter side and  $M_R$  receiver antennas in the receiver side. The block diagram is shown in figure 1. The transmitted matrix is a  $M_T \times 1$  column matrix  $\mathbf{s}$ , where  $s_i$  is the  $i$ th component transmitted from the antenna  $i$ . We assume a Gaussian channel such that the elements of  $\mathbf{s}$  are considered to be independent identically distributed (i.i.d) Gaussian variables.

The channel matrix  $\mathbf{H}$  is a  $M_R \times M_T$  complex matrix. The component  $h_{ij}$  of the matrix is the fading coefficient from the  $j$ th transmit antenna to the  $i$ th receive antenna. We assume that the received power for each of the receive antennas is equal to the total transmitted power  $E_s$ . We assume that the channel matrix is known at the receiver but unknown at the transmitter.

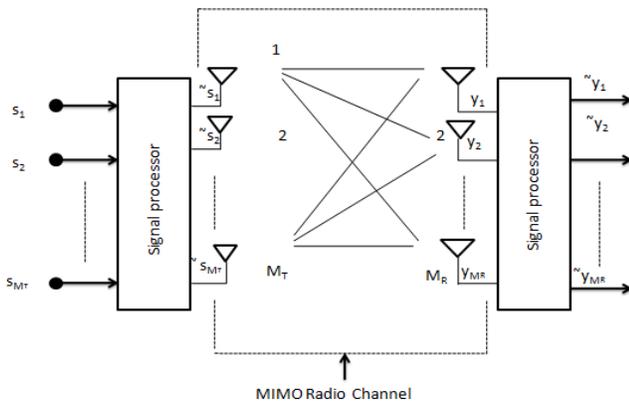
The channel matrix can be estimated at the receiver by transmitting a training sequence. If we require the transmitter to know this channel, then we need to

communicate this information to the transmitter via a feedback channel. The elements of  $\mathbf{H}$  can be deterministic or random. The noise at the receiver is another column matrix of size  $M_R \times 1$ , denoted by  $\mathbf{n}$ . The components of  $\mathbf{n}$  are zero mean circularly symmetrical complex Gaussian (ZMCSCG) variables. Each of the  $M_R$  receive branches has identical noise power of  $N_0$ . The receiver operates on the maximum likelihood detection principle over  $M_R$  receive antennas. The received signals constitute a  $M_R \times 1$  column matrix denoted by  $\mathbf{r}$ , where each complex component refers to a receive antenna. Since we assumed that the total received power per antenna is equal to the total transmitted power, the SNR can be written as

$$\gamma = Es / N_0. \tag{1}$$

Hence, the received vector can be written as,

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{2}$$



### III. CAPACITY OF DIFFERENT ANTENNA CONFIGURATIONS COMPARED TO MIMO

The capacity of different antenna configurations like SISO, SIMO, and MISO are compared with MIMO. The capacity of MIMO is found to be better than all the other configurations. The capacity of the different configurations is shown below:

#### A. Single-Input, Single-Output (SISO)

This is the conventional system that is used everywhere constituting one antenna on either end of the wireless link. For a given channel, we assume that the bandwidth is  $B$ , and a given transmitter power of  $P$  the signal at the receiver has an average signal-to-noise ratio of  $SNR$ . Then, an estimate for the Shannon limit on channel capacity,  $C$ , is

$$C \sim B. \log_2 (1+SNR) \tag{3}$$

#### B. Single Input Multiple Output(SIMO)

For the SIMO system, we have single antenna at the transmitter and  $M_R$  antennas at the receiver. If the

signals received on these antennas have the same amplitude on average then they can be coherently added to produce an increase in the signal power. Since there are  $M_R$  sets of noise that are added incoherently it results in an  $M_R$  fold increase in the noise power. Hence, there is an overall increase in the SNR. Thus, the channel capacity for this channel is approximately equal to

$$C \sim B. \log_2 (1+ M_R. SNR) \tag{4}$$

#### C. Multiple Input Single Output(MISO)

In the MISO system, we have  $M_T$  transmitting antennas and a single receiving antenna. The total transmitted power is divided up into the  $M_T$  transmitter branches. Following a similar argument as for the SIMO case, if the signals add coherently at the receiving antenna we get approximately an  $M_T$ -fold increase in the SNR as compared to the SISO case. Note here, that because there is only one receiving antenna the noise level is the same as in the SISO case.

Thus, the channel capacity is approximately given as

$$C \sim B. \log_2 (1+ M_T . SNR) \tag{5}$$

#### D. Multiple Input Multiple Output(MIMO)

The MIMO system can be viewed in effect as a combination of the MISO and SIMO channels. In this case, it is possible to get approximately an  $M_T \times M_R$  -fold increase in the SNR yielding a channel capacity equal to

$$C \sim B. \log_2 (1+ M_T . M_R . SNR) \tag{6}$$

Thus, we can see that the channel capacity for the MIMO system is higher than that of MISO or SIMO. However, we should note here that in all four cases the relationship between the channel capacity and the SNR is logarithmic. This means that trying to increase the data rate by simply transmitting more power is extremely costly.

### IV. COMMON RECEIVER ALGORITHMS

Several MIMO receiver algorithms are proposed in the literature. In this paper, a variety of these techniques will be evaluated using predetermined performance and complexity criterion. Some of the algorithms are discussed below.

#### A. Zero Forcing(ZF)

Zero Forcing is one of the linear detection techniques which linearly filter the received signals using linear filter matrices and independently decodes them. ZF can be implemented using the inverse of the channel matrix  $\mathbf{H}$  (assumed to be invertible) to obtain the estimate of the transmitted vector  $\mathbf{s}$ .

$$\begin{aligned} \tilde{\mathbf{s}} &= \mathbf{H}^\dagger \mathbf{r} \\ &= \mathbf{H}^\dagger (\mathbf{H}\mathbf{s} + \mathbf{n}) \\ &= \mathbf{s} + \mathbf{H}^\dagger \mathbf{n} \end{aligned}$$

With the addition of the noise vector, ZF estimate,  $\tilde{\mathbf{s}}$  consists of the decoded vector plus a combination of the inverted channel matrix and the unknown noise vector. Because the pseudo inverse of the channel matrix may have

high power when the channel matrix is ill-conditioned, the noise variance is consequently increased and the performance is degraded. To alleviate the noise enhancement introduced, the MMSE detector was proposed, where the noise variance is considered in the construction of the filtering matrix.

**B. Minimum Mean Square Error (MMSE)**

Minimum Mean Square Error (MMSE) approach alleviates the noise enhancement problem by taking into consideration the noise power when constructing the filtering matrix. The vector estimates produced by an MMSE filtering matrix becomes

$$\hat{s} = [(H^H H + (\sigma^2 I))^{-1}] H^H r \tag{7}$$

where  $\sigma$  is the noise variance. The added term ( $1/SNR = \sigma^2$ , in case of unit transmit power) offers a trade-off between the residual interference and the noise enhancement. Namely, as the SNR grows large, the MMSE detector converges to the ZF detector, but at low SNR it prevents the worst Eigen values from being inverted. At low SNR, MMSE becomes a Matched filter.

$$[(H^H H + (\sigma^2 I))^{-1}] H^H = \sigma^2 H^H \tag{8}$$

The MMSE receiver, on the other hand, can minimize the overall error caused by noise and mutual interference between the co-channel signals, but this is at the cost of separation quality of the signals.

**C. VBLAST Receivers**

Although linear detection techniques are easy to implement, they lead to high degradation in the achieved diversity order due to the linear filtering. Another approach that takes advantage of the diversity potential of the additional receive antennas, uses nonlinear techniques such as Successive cancellation (SIC). A new symbol detection algorithm called VBLAST/MAP (Maximum a posteriori Probability) that has a layered structure as VBLAST was introduced. This gives a better error performance at a slightly higher complexity. For the signal detection problem, (MAP) is defined as

$$\tilde{s} = \arg \max_{s \in A^{M_T}} \{Pr (s| r \text{ is received})\} \tag{9}$$

where  $A$  is the modulation alphabet, mostly QAM alphabet. This detection algorithm is a recursive procedure that extracts the components of the transmitted vector  $s$  according to a certain ordering ( $k_1, k_2 \dots k_{M_T}$ ) of the indices of the elements of 'a'. Thus, ( $k_1, k_2 \dots k_{M_T}$ ) is a permutation depends on  $H$  but not on the received vector  $r$ .

**D. Successive Cancellation Algorithm (SUC)**

This algorithm provides improved performance at the cost of increased complexity. This method first detects the first row of the signal and then cancels its effect from the overall received signal. It then proceeds to the next row. Now the channel matrix has a reduced dimension of  $M_R \times (M_T - 1)$

and a signal dimension of  $(M_T - 1) \times 1$ . Then it does the same operation on the next row.

**E. Maximum Likelihood Receiver**

This is an optimum receiver. If the data stream is temporally uncoded, the ML receiver solves

$$s^{\wedge} = \arg \min \|r - Hs\|^2 \tag{10}$$

where  $s^{\wedge}$  is the estimated symbol vector. The ML receiver searches through the entire vector constellation for the most probable transmitted signal vector. This implies investigating  $S^{M_T}$  combinations, a very difficult task. Hence, these receivers are difficult to implement, but provide full  $M_R$  diversity and zero power losses as a consequence of the detection process. In this sense it is optimal. There have been developments based on fast algorithms employing sphere decoding.

**F. Sphere Decoding Algorithm (SD)**

In this section, the basics of SD are briefly reviewed, and outline the corresponding state of the art.

**A. Sphere Constraint**

The main idea in SD is to reduce the number of candidate vector symbols to be considered in the search that solves (10) without accidentally excluding the ML solution. This goal is achieved by constraining the search to only those points of  $Hs$  that lies inside a hyper sphere with radius  $h$  around the received point  $r$ . The corresponding inequality is referred to as the sphere constraint (SC):

$$d(s) < h^2 \quad \text{with} \quad d(s) = \|r - Hs\|^2 \tag{11}$$

**B. Tree Pruning**

Only imposing the SC does not lead to complexity reductions as the challenge has merely been shifted from finding the closest point to identifying points that lie inside the sphere. Hence, complexity is only reduced if the SC can be checked other than again exhaustively searching through all possible vector symbols. Two key elements allow for such a computationally efficient solution:

*1) Computing Partial Euclidean Distances:* We start by noting that the channel matrix  $H$  can be triangularized using a QR decomposition according to  $H = QR$ , where the  $M_R \times M_T$  matrix  $Q$  has orthonormal columns (i.e.,  $Q^H Q = I_{M_T}$ ), and the  $M_T \times M_T$  matrix  $R$  is upper triangular. It can easily be shown that

$$d(s) = c + \|f - Rs\|^2 \quad \text{with} \quad f = Q^H y = R s^{ZF}$$

where  $s^{ZF}$  is the zero-forcing (or unconstrained ML) solution  $s^{ZF} = H^{\dagger} y$ . The constant  $c$  is independent of the vector symbol  $s$  and can hence be ignored in the metric computation. In the following, for simplicity of exposition,

we set  $c = 0$ . If we build a tree such that the leaves at the bottom correspond to all possible vector symbols  $s$  and the possible values of the entry  $s_{MT}$  define its top level, we can uniquely describe each node at level  $i$  ( $i = 1, 2, \dots, M_T$ ) by the partial vector symbols  $s^{(i)} = [s_i, s_{i+1}, \dots, s_{M_T}]^T$  BPSK modulation. Now, we can recursively compute the (squared) distance  $d(s)$  by traversing down the tree and effectively evaluating  $d(s)$  in a row-by-row fashion:  
 We start at level  $i = M_T$  and set  $T_{M_T+1}^{(s^{(M_T+1)})} = 0$ . The partial (squared) Euclidean distances (PEDs)  $T_i^{(s^{(i)})}$  are then given by

$$T_i^{(s^{(i)})} = T_{i+1}^{(s^{(i+1)})} + |e_i^{(s^{(i)})}|^2 \quad (12)$$

with  $i = M_T, M_T - 1, \dots, 1$ , where the distance increments  $|e_i^{(s^{(i)})}|^2$  can be obtained as

$$|e_i^{(s^{(i)})}|^2 = \left| r_i - \sum_{j=i}^{M_T} R_{ij} s_j \right|^2 \quad (13)$$

Finally,  $d(s)$  is the PED of the corresponding leaf:  $d(s) = T_1^{(s)}$ . Since the distance increments are  $|e_i^{(s^{(i)})}|^2$  nonnegative, it follows immediately that whenever the PED of a node violates the (partial) SC given by

$$T_i^{(s^{(i)})} < r^2 \quad (14)$$

the PEDs of all its children will also violate the SC. Consequently, the tree can be pruned above this node. This approach effectively reduces the number of vector symbols (i.e., leaves of the tree) to be checked.

2) *Tree Traversal and Radius Reduction*: When the tree traversal is finished, the leaf with the lowest  $T_1(s)$  corresponds to the ML solution. The traversal can be performed breadth-first or depth-first. In both cases, the number of nodes reached and hence the decoding complexity depends critically on the choice of the radius  $r$ . The K-best algorithm approximates a breadth-first search by keeping only (up to)  $K$  nodes with the smallest PEDs at each level. The advantage of the K-best algorithm over a full (depth-first or breadth-first) search is its uniform data path and a throughput that is independent of the channel realization and the SNR. However, the K-best algorithm does not necessarily yield the ML solution. In a depth-first implementation, the complexity and dependence of the throughput on the initial radius can be reduced by shrinking the radius  $r$  whenever a leaf is reached. This procedure does not compromise the optimality of the algorithm, yet it decreases the number of visited nodes compared to a constant radius procedure. As an added advantage of the depth-first approach with radius reduction, the initial radius may be set to infinity, alleviating the problem of initial radius choice. However, in contrast to the K-best algorithm, a depth-first traversal does not yield a deterministic throughput.

### V. SIMULATION RESULTS

The capacity increase in case of MIMO when compared to all the other configurations are simulated using MATLAB. (fig.7).The increase in capacity with various antenna configurations in MIMO are also plotted in fig.8.All the algorithms stated in section IV was simulated in MATLAB and ML was found to be the best in terms of BER performance. But the complexity of ML increases as the number of transmitters increase. Here a  $4 \times 4$  and a  $2 \times 2$  systems are implemented both using PSK and QAM modulation. For VBLAST we have taken  $8 \times 12$  system. The results have been extended to MIMO –OFDM also (fig.9).

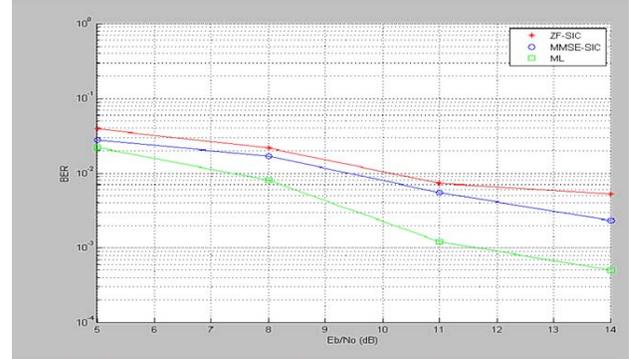


Fig.2 BER performance of ZF, MMSE and ML algorithms using 2x2 QPSK modulation

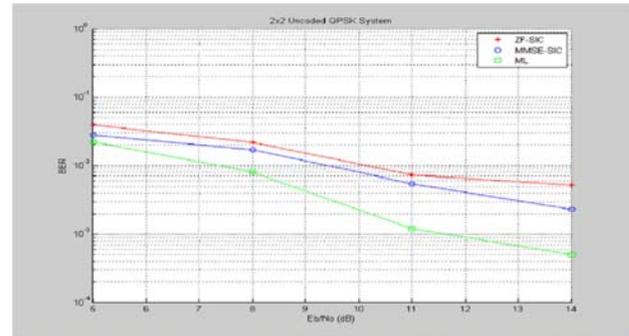


Fig.3 BER performance of ZF, MMSE and ML algorithms using 2x2 QPSK modulation

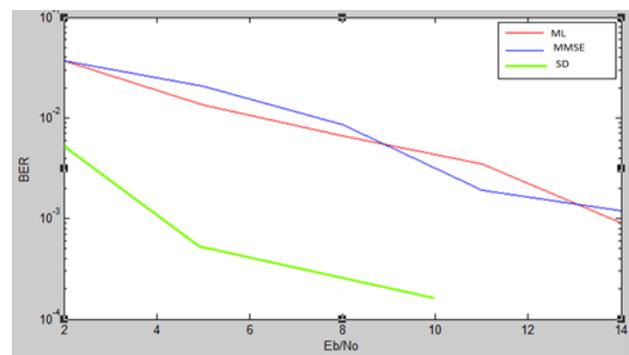


Fig.4 BER performance of SD, MMSE and ML algorithms using 4x4 QAM modulation

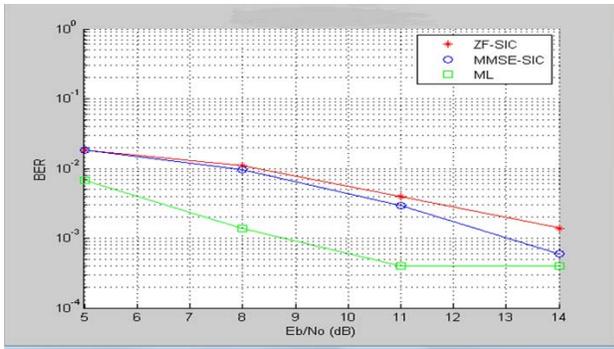


Fig.5 BER performance of ZF, MMSE and ML algorithms using 4x4 QPSK modulation

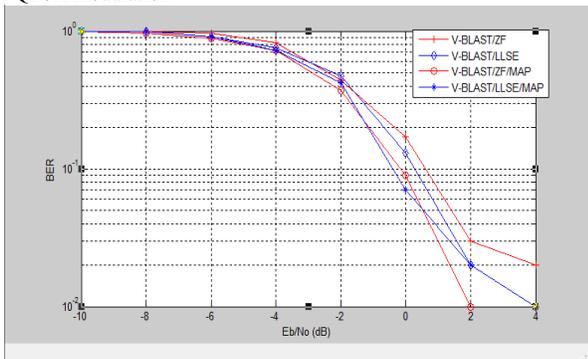


Fig.6 BER performance of the combined VBLAST algorithms using 8x12, 16-QAM modulation

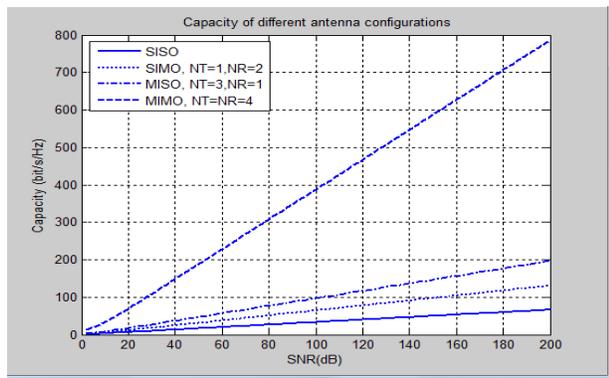


Fig.7 Capacity of different antenna configurations-SISO, SIMO, MISO, MIMO

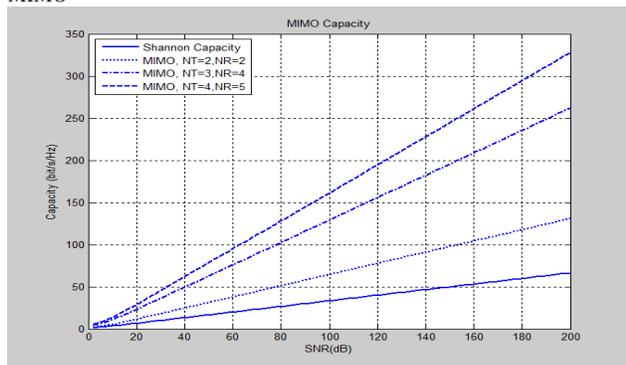


Fig.7 Capacity of different MIMO configurations-with different number of transmitters and receivers.

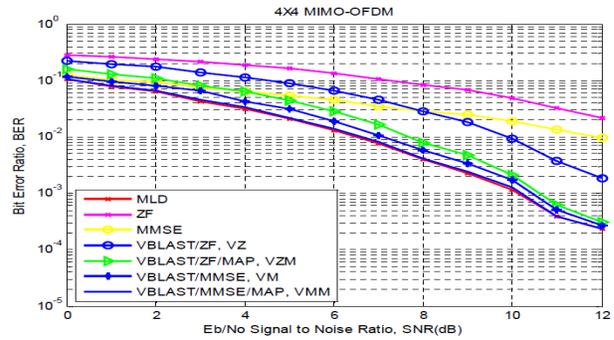


Fig 9. BER Performance of the detection algorithms in MIMO - OFDM using 4x4 system.

VI. CONCLUSION

The MATLAB results we can see that the capacity of MIMO is very high when compared with all the other configurations. The capacity of MIMO increases when the number of transmitters and receivers increase.

From the MATLAB results we can see that ML has got a better performance in terms of BER. But the complexity increases as the number of transmitters increase. The Sphere Decoding algorithm is a solution for this problem. This reduces the complexity of ML.

VII. FUTURE WORKS

Our future work will be mainly concentrating on a new Sphere decoding algorithm which will reduce the hardware complexity and to develop a rapid prototype of MIMO based on it, interlinking with the FPGA.

REFERENCES

- [1] Mohinder Jankiraman, *Space Time Codes and MIMO Systems*, Universal personal communication, Aston house, Boston, London, 2004.
- [2] Jingming Wang, Babak Daneshrad, "A Comparative Study of MIMO Detection Algorithms for Wideband Spatial Multiplexing Systems," *IEEE Communications Society / WCNC 2005*, pp.408-413.
- [3] Mohamed Oussama Damen, Hesham El Gamal, Giuseppe Caire, "On Maximum-Likelihood Detection and the Search for the Closest Lattice Point," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp 2389-2402, October 2003.
- [4] Luis G. Barbero, and John S. Thompson, " *Rapid Prototyping System for the Evaluation of MIMO Receive Algorithms*", EUROCON 2005, Serbia and Montenegro Belgade, November 22-24, 2005.
- [5] A. Burg, M. Borgmann, M. Wenk, M. Zellweger, W. Fichtner and H. Bolcskei. "VLSI Implementation of MIMO Detection Using the Sphere Decoding Algorithm", *IEEE Journal of Solid-State Circuits*, volume 40(7):pp. 1566-1577, July 2005.
- [6] Yavuz Yapici, "V-BLAST/MAP: A New symbol detection algorithm for MIMO channels", MS in Electrical and Electronics Engineering, Bilkent University, January 2005.
- [7] Z. Guo and P. Nilsson, "A VLSI implementation of MIMO detection for future wireless communications," in *Proceedings. IEEE PIMRC*, volume. 3, 2003.
- [8] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Transactions on Information Theory*, vol.44, pp. 744-765, Mar. 1998.