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Fermatean Fuzzy Multi Group over Multi-Homomorphisms

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Abstract: The concept of the fermatean fuzzy multi group offers a useful technique for real life transportation problems. This special phenomenon is used to model the structure of the controlling an intersection of two-way sheets. In this paper, we study the concept of Λ – fermatean fuzzy multi group structures, Λ – fermatean fuzzy multi cossets and Λ – fermatean fuzzy multi normal subgroup structures. Finally, we define Λ – fermatean fuzzy multi homomorphism between any two Λ – fermatean fuzzy multi groups and establish some important properties of this phenomenon.

Keywords: Fuzzy set, multi set. multi group, fermatean fuzzy multi set, normal subgroup, cossets, homomorphism, image.

1.INTRODUCTION:

The theory of collections is a necessary mathematical tool. It gives mathematical models for the class of problems that explains with exactness, precision and uncertainty. Characteristically, non crisp set theory is extensional. More often than not, the real life problems inherently involve uncertainties, imprecision and not clear. In particular, such classes of problems arise in economics, engineering, environmental sciences, medical sciences, and social sciences etc. Zadeh [13] defined fuzzy set theory in his pioneering paper in 1965. In order to solve various types of uncertainties and complex MAGDM problems, the theory of fuzzy sets is proposed by Zadeh [13]. Later on, Atanassov [1] introduced the intuitionistic fuzzy set (IFS) theory to extend the concept of fuzzy set. Yager [12] explored a typical division of these collections known as q-rung orthopair uncertainty collection in which the aggregate of the q^{th} power of the help for and the q^{th} power of the help against is limited by one. He explained that as 'q' builds the space of truth able orthopairs increments and therefore gives the user more opportunity in communicating their conviction about value of membership. At the point when q = 3, Senapathi and Yager [6] have evoked q-rung orthopair uncertainty collection as fermatean uncertainty sets (FUSs).Pythagorean uncertainty collections have studied the concentration of many researchers within a short period of time. For example, Yager [10] has derived up a helpful decision technique in view of pythagoreanun certainty aggregation operators to deal with pythagoreanun certainty MCDM issues. Yager and Abbasov [11] studied the pythagorean membership grades (PMGs) and the considerations related to Pythagorean uncertainty collections and presented the association between the PMGs and the imaginary numbers. Senapathi and Yager [6] specified basic activities over the FUSs and concentrated new score mappings and accuracy mappings of FUSs. They proposed the technique for order preference by similarity to ideal Solution (TOPSIS) way to deal with taking care of the issue with fermatean uncertainty data. In an attempt to model uncertainty, the notion of fuzzy sets was proposed by zadeh [13] as a method for representing imprecision in real-life situations. One can say, a fuzzy set (or a fuzzy subset of a set) is the fuzzification of crisp set to capture uncertainty in a collection. The concept of fuzzy set has grown stupendously over the years giving birth to fuzzy group which is the application of fuzzy sets to the elementary theory of groups and groupoids as noted in [5]. Several works has been done on fuzzy groups since inception; some could be found in [3]. Motivated by zadeh [13], the idea of fuzzy multisets was introduced in [3] as the generalization of fuzzy sets or the fuzzification of multi sets in [5]. Recently, Shinoj et al. [8] followed the foot steps of Rosenfeld [6] and introduced a non-classical group called fuzzy multi group. In particular, the idea of fuzzy multigroups generalized fuzzy groups. In this paper, we study the concept of Λ – fermatean fuzzy multi group structures, Λ – fermatean fuzzy multi cossets and Λ – fermatean fuzzy multi normal subgroup structures. Finally, we define Λ – fermatean fuzzy multi homomorphism between any two Λ – fermatean fuzzy multi groups and establish some important properties of this phenomenon.

2. PRELIMINARIES:

Definition 2.1: Let X be a set. A multi set M is characterized by a count function C_M: X →N , when N = N∪{0}. For each. x ∈ X, $C_M(x)$ is the characteristic value of x in M. The set of all multi sets of X is denoted by $M_S(X)$.

Definition 2.2: Let X be a group. A multi set M is called a multi group of X if it satisfies the following conditions;

(i) $C_M(xy) \geq \min \{ C_M(x), C_M(y)\}$

(ii) $C_M(x^{-1}) \geq C_M(x)$, for all x ∈ X. we denote the set of all multi groups of X by $M_G(X)$.

Definition 2.3: If X is a collection of objects, then a fuzzy set A in X is a set of ordered pairs; A = { (x , $\mu_A(x)$ / x ∈ X, μ_A : X → [0,1] ,where , μ_A is called the membership function of A, and is defined from X into [0, 1].

Definition 2.4: [Senapati and Yager, 2019a] Let 'X' be a universe of discourse A. Fermatean uncertainty set "F" in X is an object having the form $F = \{\langle x, m_F(x),\ n_F(x)\rangle / x \in X\}$, where $m_F(x): X \to [0,1]$ and $n_F(x): X \to [0,1]$, including the condition $0 \leq \left(m_F(x)\right)^3 + \left(m_F(x)\right)^3 \leq 1, for\ all\ x \in X$.The numbers $m_F(x)$ signifies the level (degree) ofmembership and $n_F(x)$ indicate the non-membership of the element ‘x’ in the set F.All through this paper, we will indicate a fermatean uncertainty set is FUS. For any FUS ‘F’ and $x \in X$, $\pi_F(x) = \sqrt[3]{1-(m_F(x))^2 - (n_F(x))^2}$ is to find out as the degree of indeterminacy of ‘x’ to F. For convenience, Senapathi and Yager called $(m_F(x),\ n_F(x))$ a fermatean uncertaintynumber (FUN) denoted by $F = (m_F,\ n_F)$.

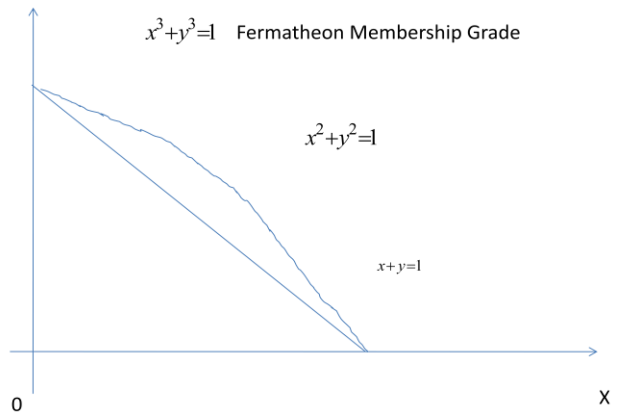

Fig-1

We will explain the membership grades (MG’s) related Fermatean uncertainty collections as fermatean membership grades.

Theorem 2.5:[Senapati and Yager, 2019a] The collections of FMG’s is higher than the set of Pythagorean membership grades (PMG’s) and bi uncertainty membership grades (BMG’s).

Proof: This improvement can be evidently approved in the following figure.

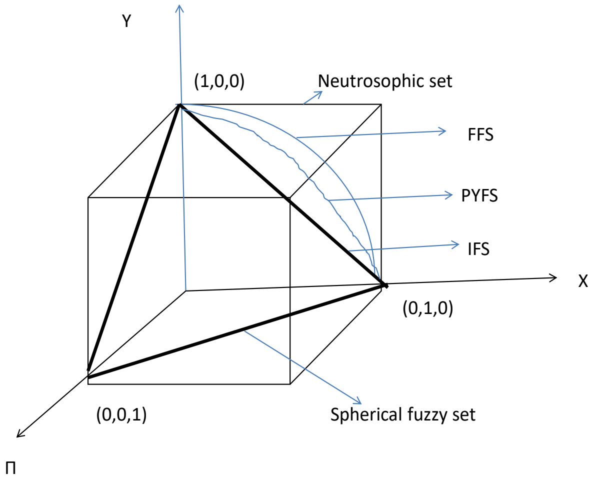

Fig-2

Here we find that BMG’s are all points beneath the line $x + y \leq 1$, the PMG’s are all points with $x^2 + y^2 \leq 1$. We see that the BMG’s enable the presentation of a bigger body of non-standard membership grades than BMG’s and PMG’s. For convenience ($C_{MF}(x)$, $C_{NF}(x)$) is called fermatean fuzzy multi number (FFMN) denoted by ($C_{MF}(x)$, $C_{NF}(x)$). A Fermatean fuzzy multi sets (FFMS) F is denoted by

F = { < x , ($\mu_A^1(x)$ (x), $\mu_A^2(x)$, …………, $\mu_A^p(x)$, $v_A^1(x)$, $v_A^2(x)$,…………, $v_A^p(x)$ > / x ∈ X}.

Remark 2.6: we arrange the membership sequence in decreasing order but the corresponding non- membership sequence may not be in decreasing or increasing order.

Definition 2.7: Let X be a group. A FFMS ‘F’ is called a fermatean fuzzy multi group of X if it satisfies the condition

(i) $C_{MF}(xy) \geq \min\{C_{MF}(x), C_{MF}(y)\}$and $C_{NF}(xy) \leq \max\{C_{NF}(x), C_{NF}(y)\}$

(ii) $C_{MF}(x^{-1}) \geq C_{MF}(x)$ and $C_{NF}(x^{-1}) \leq C_{NF}(x)$, for all x ∈ X.

We denote the set of all fermatean fuzzy multi groups of X by FFMG(X).

Example 2.8: Let F_1 = { < 0.7 , 0.3 / x >, < 1 , 0.7 / y >, < 0.2 , 0.7 / z >} and F_2 = { < 0.8 , 0.5 / x >, < 0.7 , 0.9 / y >, < 0.1 , 0.4 / z > } for X = { x,y,z}. Then Clearly F_1 and F_2 are fermatean fuzzy multi group of X.

Example 2.9: Assume that X = {a ,b, c} is a set. Then for $C_{MF}(a)$ = { 1,0.5. 0.7}, $C_{MF}(b)$ = { 0.3. 0.7}, $C_{MF}(c)$ = { 0}, $C_{NF}(a)$ = { 0.2,0.5. 0.7}, $C_{NF}(b)$ = { 1,0.7}, $C_{NF}(c)$ = { 0.3},

A FFMS of X written as F = { < 1, 0.5, 07 / a > , < 0.3, 07 / b >, < 0.2, 0.5, 07 / a >, < 0.3 / c >.

Definition 2.10: Let C be a non-empty crisp set and $\mathcal{F}_1 = (\mu^3_1(m), \upsilon^3_1(m))$ and $\mathcal{F}_2 = (\mu^3_2(m), \upsilon^3_2(m))$ be FFSs on C. Then

(i) $\mathcal{F}_1 = \mathcal{F}_2$ if and only if $\mu^3_1(m) \leq \mu^3_2(m)$ and $\upsilon^3_1(m) \geq \upsilon^3_2(m)$.

(ii) $\mathcal{F}_1 = \mathcal{F}_2$ if and only if $\mathcal{F}1 \subset \mathcal{F}2$ and $\mathcal{F}2 \subset \mathcal{F}1$.

(iii) $\mathcal{F}^c_1 = (\upsilon^3_1(m), \mu^3_1(m))$.

(iv) $\mathcal{F}1 \cap \mathcal{F}2$ = (min { $\mu^3_1(m)$,$\mu^3_2(m)$}, max {$\mu^3_1(m)$,$\mu^3_2(m)$})

(v) $\mathcal{F}1 \cup \mathcal{F}2$ = (max { $\mu^3_1(m)$,$\mu^3_2(m)$}, min {$\mu^3_1(m)$,$\mu^3_2(m)$})

(vi) [ ] $\mathcal{F}1$ = ($\mu^3_1(m)$,1 - $\mu^3_2(m)$}, < > $\mathcal{F}1$ = (1 - $\upsilon^3_2(m)$, $\upsilon^3_1(m)$).

Definition 2.11: Let $F_1, F_2 \in$ FFMS(X). Then F_1 is called a fermatean fuzzy submulti set of F_2 written as $F_1 \subseteq F_2$ if $C_{MF1}(x) \leq C_{MF2}(x)$, $C_{NF1}(x) \leq C_{NF2}(x)$, for all x ∈ X.

Definition 2.12: Let α and β be positive real numbers lie in the closed unit interval such that $0 \leq \alpha + \beta \leq 1$.Then (α , β)-cut set of FFMS ‘F’ of the universe ‘U’ is a crisp set consisting of all these elements of U for which $C_{MF}(x) \geq \alpha$ and $C_{NF}(x) \leq \beta$ for all x ∈ U.

Remark 2.13: A FFMS ‘F’ of a group is FFMG if each of its (α , β)-cut set is a subgroup of G.

Definition 2.14: A FFMS ‘F’ is said to be fermatean fuzzy multi normal subgroup (FFMNG) if it meets the following conditions;

(i) $C_{MF}(xy) = C_{MF}(yx)$ and (ii) $C_{NF}(xy) = C_{NF}(yx)$, for all x, y ∈ G.

Definition 2.15: Let F_1 and F_2 be two FFMS's of the universe U. Then the average operator F_1 \$ F_2, is defined as $F_1\$F_2 = \{ < s_1 ,\sqrt{C_{MF1}(x)\, C_{MF2}(x)} , \sqrt{C_{NF1}(x)\, C_{NF2}(x)} > / x \in U\}$.

3. Properties Of Λ – Fermatean Fuzzy Multi Group

In this section, we study Λ – fermatean fuzzy multi subgroup. Moreover, numerous useful results and algebraic properties are introduced.

Definition 3.1: Suppose F is a FFMS of a universe S and $\Lambda \in [0, 1]$. Then FFMS

$F^{\Lambda} = (C_{MF}^{\Lambda} , C_{NF}^{\Lambda})$ is called Λ – FFMS of universe S with respect to FFMS 'F'; where

$C_{MF}^{\Lambda}(x) = \text{₰} (C_{MF}(x) , \Lambda)$, $C_{NF}\Lambda(x) = \Omega (C_{NF}(x) , 1-\Lambda)$ and ₰ , Ω denote the average operator defined in 2.15.

Proposition 3.2: Let F_1 and F_2 be two Λ – FFMS's of the universe U. Then so is $F_1^{\Lambda} \cap F_2^{\Lambda}$.

Proof: Consider

$C_{M(F1\cap F2)}^{\Lambda}(x) = \text{₰} \{ C_{M(F1\cap F2)}(x) , \Lambda \} = \text{₰} \{ \min \{ C_{MF1}(x) , C_{MF2}(x)\} , \Lambda \}$

$= \text{₰} \{ \min \{ C_{MF1}(x) , \Lambda\} , \min \{C_{MF2}(x) , \Lambda \}\}$

$= \min \{ C_{MF1}^{\Lambda}(x) , \min \{C_{MF2}^{\Lambda}(x) \}\}= C_{M(F1^{\Lambda} \cap F2^{\Lambda})}$for all $x \in U$.

Similarly, it can be proved that

$C_{N(F1\cap F2)}^{\Lambda}(x) = C_{N(F1^{\Lambda} \cap F2^{\Lambda})}(x)$, for all $x \in U$.Hence $(F_1\cap F_2)^{\Lambda} = (F_1^{\Lambda}\cap F_2^{\Lambda})$.

Remark 3.3: The union of any two Λ – FFMS's is also Λ – FFMS.

Definition 3.4: A FFMS of a group G is 'Λ – FFMS's 'F^{Λ}' satisfying the following condition (i) $C_{MF}^{\Lambda}(xy) \geq \min \{ C_{MF}^{\Lambda}(x), C_{MF}^{\Lambda}(y)\}$and $C_{NF}^{\Lambda}(xy) \leq \max \{ C_{NF}^{\Lambda}(x), C_{NF}^{\Lambda}(y)\}$ (ii) $C_{MF}^{\Lambda}(x^{-1}) \geq C_{MF}^{\Lambda}(x)$ and $C_{NF}(x^{-1}) \leq C_{NF}(x)$, for all x, y ∈ G.

Remark 3.5 : Let 'e' be an identity element of G. Then $F^{\Lambda}(x) \leq F^{\Lambda}(e)$ for all x ∈ G. Also, $F^{\Lambda}(xy^{-1}) = F^{\Lambda}(e)$ implying that $F^{\Lambda}(x) = F^{\Lambda}(y)$, for all y ∈ G.

Proposition 3.6: Every fermatean fuzzy multi group of G is an Λ – fermatean fuzzy multi group of G.

Proof: Let x,y ∈ G.Then by using the fact that F is fermatean fuzzy multi group. we have

$C_{MF}^{\Lambda}(xy) = \text{₰} \{ C_{MF}(xy) , \Lambda \} \geq \text{₰} \{ \min \{C_{MF}(x) , C_{MF}(y) , \Lambda \}$

$= \text{₰} \{ \min \{ C_{MF}(x) , \Lambda\} , \min \{C_{MF}(y) , \Lambda \}$

$= \min \{ C_{MF}^{\Lambda}(x) , C_{MF}^{\Lambda}(y) \}$

Similarly it can be proved that

$C_{NF}^{\Lambda}(xy) \leq \max \{ C_{NF}^{\Lambda}(x) , C_{NF}^{\Lambda}(y) \}$.

Moreover,

$C_{MF}^{\Lambda}(x^{-1}) = \text{₰} \{C_{MF}(x^{-1}) , \Lambda \} = \text{₰} \{C_{MF}(x) , \Lambda \} = C_{MF}^{\Lambda}(x)$.

Similarly,

$C_{NF}^{\Lambda}(x^{-1}) = \text{₰} \{C_{NF}(x-1) , \Lambda \} = \text{₰} \{C_{NF}(x) , \Lambda \} = C_{NF}^{\Lambda}(x)$.

Consequently, F^{Λ} is an Λ – fermatean fuzzy multi group of group G.

Remark 3.7: An Λ – fermatean fuzzy multi group need not be a fermatean fuzzy multi group, that is the converse of proposition 3.6 does not hold.

The above fact can be explained in the following example.

Example 3.8: Let G = {e,a,b,c } be the Kelin's -4 group. we define FFMS F of G as

F = { < e, 0.4 ,05 > , < a, 0.3 ,0.5> , < b, 0.2, 0.8 > , < c ,0.1,0.7 > } Note that, F is not FFMG of G. Let Λ = 0.5. Then F^{Λ} = { < e,0.4 ,05 > , < a, 0.3 ,0.5> , < b, 0.2, 0.8 > , < c ,0.1,0.7 > }.It is clear that (0.4 , 0.5)-cut set of 0.5 .FFMS is $\Lambda^{0.5}$ = {a} and $\Lambda^{0.6}$ = {a} , whereas (0.5 , 0.8)-cut set of 0.5. FFMS is given by $\Lambda^{0.5}$ = {e , a} and $\Lambda^{0.6}$ = {e, a, b, c}.

Note that, each of the above cut set of Λ – fermatean fuzzy multi set is a subgroup of G. Hence it is an Λ – fermatean fuzzy multi group.

The following results presents the condition under which a given Λ – fermatean fuzzy multi group is an Λ – fermatean fuzzy multi group.

Proposition 3.9: Let F be any FFMS of a group G such that $C_{MF}(x^{-1}) = C_{MF}(x)$ and $C_{MF}(x^{-1}) = C_{MF}(x)$ for all x ∈ G. Moreover , Λ < min { a, 1-b }, where a = min $\{C_{MF}(x) / x \in G\}$ and b = max $\{C_{NF}(x) / x \in G\}$, then F is an Λ – fermatean fuzzy multi group of G.

Proof: In view of given conditions, we have a > Λ and b < 1- Λ.

It follow that $C_{MF}(x) > \Lambda$ and $C_{NF}(x) < 1- \Lambda$, for all x ∈ G.

Therefore , $C_{MF}^{\Lambda}(xy) \geq \min \{ C_{MF}^{\Lambda}(x), C_{MF}^{\Lambda}(y)\}$ and $C_{NF}^{\Lambda}(xy) \leq \max \{ C_{NF}^{\Lambda}(x) , C_{NF}^{\Lambda}(y),\}$ for all x,y ∈ G.Moreover, for any x ∈ G , we obtain

$C_{MF}(x^{-1}) = C_{MF}(x)$ and $C_{NF}(x^{-1}) = C_{NF}(x)$. This shows that $C_{MF}^{\Lambda}(x^{-1}) = C_{MF}^{\Lambda}(x)$ and $C_{NF}^{\Lambda}(x^{-1}) = C_{NF}^{\Lambda}(x)$. The subsequent result indicates that the intersection of any two Λ – fermatean fuzzy multi group is an Λ – fermatean fuzzy multi group of G.

Proposition 3.10: The intersection of two Λ – fermatean fuzzy multi groups of a group G is also an Λ – fermatean fuzzy multi group.

Proof: Suppose F_1^{Λ} and F_2^{Λ} are Λ – fermatean fuzzy multi groups of a group G.Then

$C_{M(F1\cap F2)}^{\Lambda}(xy) = \text{₰} \{ C_{M(F1\cap F2)}(xy) , \Lambda \}= \text{₰} \{ \min \{ C_{MF1}(xy) , C_{MF2}(xy)\} , \Lambda \} = \text{₰} \{ \min \{ C_{MF1}(xy) , \Lambda\} , \min \{C_{MF2}(xy) , \Lambda \}\}= \min \{ C_{MF1}^{\Lambda}(xy) , \min \{C_{MF2}^{\Lambda}(xy) \}= C_{M(F1^{\Lambda} \cap F2^{\Lambda})}(xy)$, for all x ∈ U. $\geq \min \{ \min \{ C_{MF1}^{\Lambda}(x), C_{MF1}^{\Lambda}(x)\}, \min\{ C_{MF1}^{\Lambda}(y), C_{MF1}^{\Lambda}(y)\} = \min \{ C_{M(F1^{\Lambda} \cap F2^{\Lambda})}(x) , C_{M(F1^{\Lambda} \cap F2^{\Lambda})}(y)\}$.

Similarly, it can be proved that for all x,y ∈ U.

$C_{N(F1 \cap F2)}^{\Lambda}(xy) \leq \max \{ C_{N(F1^{\Lambda} \cap F2^{\Lambda})}(x) , C_{N(F1^{\Lambda} \cap F2^{\Lambda})}(y)\}$. Also

$C_{M(F1 \cap F2)}^{\Lambda}(x^{-1}) = \text{₰} \{C_{M(F1 \cap F2)}(x^{-1}), \Lambda \} = \text{₰} \{ \min\{C_{MF1}(x^{-1}), \Lambda\}, \min \{C_{MF2}(x^{-1}), \Lambda\}, = \min \{C_{MF1}(x^{-1}),\{C_{MF2}(x^{-1})\}= C_{M(F1^{\Lambda} \cap F2^{\Lambda})}(x)$. Similarly, $C_{N(F1 \cap F2)}^{\Lambda}(x^{-1}) = C_{N(F1^{\Lambda} \cap F2^{\Lambda})}(x)$.

Corollary 3.11: The intersection of any number of Λ – fermatean fuzzy multi groups of a group G is also an Λ – fermatean fuzzy multi group of G.

Remark 3.12: The Union of any number of Λ – fermatean fuzzy multi groups of a group G may not be an Λ – fermatean fuzzy multi group of G.

Definition 3.13: Let F^{Λ} be an Λ – fermatean fuzzy multi group of G and $x \in G$. An Λ – fermatean fuzzy multi right cosset of 'F' in G, denoted by $F^{\Lambda}x$ is defined as

$F^{\Lambda} x(g) = (C_{MF}{}^{\Lambda} x(g) , C_{NF}{}^{\Lambda}x(g))$, where $C_{MF}{}^{\Lambda}x (g) = \wp (C_{MF}(gx^{-1}) , \Lambda)$, $C_{NF}{}^{\Lambda}x(g) = \Omega (C_{NF}(gx^{-1}) , 1- \Lambda)$ for all $g \in G$. Similarly, we can define Λ – fermatean fuzzy multi left cosset of F in G.

Definition 3.14: An Λ – fermatean fuzzy multi group F^{Λ} of G is called Λ – fermatean fuzzy multi normal subgroup of G if $xF^{\Lambda} = F^{\Lambda}x$, for all $x \in G$.

The following result shows that every fermatean fuzzy multi normal subgroup of G is also Λ – fermatean fuzzy multi normal subgroup of G.

Proposition 3.15: If F is a fermatean fuzzy multi normal subgroup of a group, then F^{Λ} is also an Λ – fermatean fuzzy multi normal subgroup of G.

Proof: Let F be a fermatean fuzzy multi normal subgroup of G. Then for all $x,g \in G$.

$C_{MF} (gx^{-1}) = C_{MF} (x^{-1}g)$ and $C_{NF} (gx^{-1}) = C_{NF} (x^{-1}g)$.

Implying that $C_{MF}{}^{\Lambda}(x^{-1}g) = \wp (C_{MF}(x^{-1}g) , \Lambda) = \wp (C_{MF}(gx^{-1}) , \Lambda) = C_{MF}{}^{\Lambda} (gx^{-1})$,

Similarly, we can prove that $C_{NF} (gx^{-1}) = C_{NF} (x^{-1}g)$. Consequently, $xF^{\Lambda} = F^{\Lambda}x$.

The converse of the given result does not hold generally. This fact can be viewed in the successive example.

Example 3.16: Let $D_3 = < a,b : a^3 = b^2 = e.\ ba = a^2b >$ be dihedral group of order 6. Define fermatean fuzzy multi normal subgroup of G 'F' of D_3 by

$$C_{MF}(x) = \begin{cases} 0.63 & \text{if } x \in < 6 > \\ 0.40 & \text{otherwise and} \end{cases}$$

$$C_{NF}(x) = \begin{cases} 0.70 & \text{if } x \in < 6 > \\ 0.83 & \text{otherwise} \end{cases}$$

Then F = {< e, 0.63 , 0.70 > , < a, 0.40 ,0.83 > , < a^2, 0.40, 0.86 > , < b ,0.63,0.70 > , < ab, 0.40. 0.63 >, < a^2b, 0.63, 0.63 >}.

Since $C_{MF}(xy) = 0.40 \neq 0.63 = C_{MF}(xy)$, therefore F is not an fermatean fuzzy multi normal subgroup of G.

Next, let $\Lambda = 0.2$, then $\Lambda^{0.2}$ = { < e, 0.4 , 0.4 > , < a, 0.4 ,0.4 > , < a^2, 0.4, 0.4 > , < b ,0.4,0.4 > , < ab, 0.4. 0.4 >, < a^2b, 0.4, 0.4 >}. One can see that $\Lambda^{0.2}$ is an Λ – fermatean fuzzy multi normal subgroup of G.

In the following result, a condition for Λ – fermatean fuzzy multi subgroup of G to be Λ – fermatean fuzzy multi normal subgroup of G is established.

Proposition 3.17: Let F^{Λ} be an Λ – fermatean fuzzy multi subgroup of a group G such that

$\Lambda <$ min { a , 1- b}, where a = $= \wp (C_{MF}(x) / x \in G\}$ and b = $\Omega (C_{NF}(x) / x \in G\}$.Then F^{Λ} is an Λ – fermatean fuzzy multi subgroup of a group G.

Proof: Since $a > \Lambda$ and $b < 1- \Lambda$, therefore min { $C_{MF}(x) / x \in G\} > \Lambda$ and max { $C_{NF}(x) / x \in G\} < 1- \Lambda$. Thus $C_{MF}(x) > \Lambda$ for all $x \in G$. Also, $C_{NF}(x) < 1- \Lambda$, so

$C_{MF}{}^{\Lambda}(xg) = \wp (C_{MF}(gx^{-1}) , \Lambda) = \Theta$ and $C_{NF}{}^{\Lambda}(xg) = \Omega (C_{MF}(gx^{-1}) , 1- \Lambda) = \acute{\varnothing}$, for all $g \in G$. Similarly, $C_{M\,xF}{}^{\Lambda} (g) = \wp (C_{MF}(x^{-1}g) , \Lambda) = \Theta$ and $C_{N\,xF}{}^{\Lambda} (g) = \Omega (C_{NF}(x^{-1}g) , 1-\Lambda) = \acute{\varnothing}$.

This concludes the proof.

4. Λ – FERMATEAN FUZZY MULTI HOMOMORPHISMS

In this section, we define Λ – fermatean fuzzy multi homomorphism between any two Λ – fermatean fuzzy multi groups and establish some important properties of this phenomenon.

Defnition 4.1: Let $F_1{}^{\Lambda}$and $F_2{}^{\Lambda}$ be two Λ – fermatean fuzzy multi groups of the groups G_1 and G_2 respectively and $\varsigma : G_1 \rightarrow G_2$ be a group homomorphism from $F_1{}^{\Lambda}$ to $F_2{}^{\Lambda}$ if $\varsigma (F_1{}^{\Lambda}) = F_2{}^{\Lambda}$.

The following result indicates that an Λ – fermatean fuzzy homomorphic image of the Λ – fermatean fuzzy multi group is an Λ – fermatean fuzzy multi group.

Theorem 4.2: Let F^{Λ} be two Λ – fermatean fuzzy multi group of the group G and $\varsigma : G_1 \rightarrow G_2$ be a surjective homomorphism. Then $\varsigma (F^{\Lambda})$ is an Λ – fermatean fuzzy multi group of G.

Proof: In view of the given condition , for any two elements $p, q \in G_2$, there exists $x ,y \in G$, such that $\varsigma (x) = p$ and $\varsigma (y) = q$.

Consider $\varsigma (F)^{\Lambda} (pq) = (C_{M\,\varsigma (F)}{}^{\Lambda} (pq) , C_{M\,\varsigma (F)}{}^{\Lambda} (pq))$. Which implies that $C_{M\,\varsigma (F^{\Lambda})} (pq) = C_{M\,\varsigma (F^{\Lambda})}(pq) = \wp (C_{M\,\varsigma (F)} (\varsigma (x) \varsigma (y) , \Lambda) = \wp (C_{M\,\varsigma (F)} (\varsigma (xy) , \Lambda) \geq \wp (C_{M\,F} (xy) , \Lambda) = C_{MF}{}^{\Lambda} (xy) \geq \min \{ C_{MF}{}^{\Lambda}(x) , C_{MF}{}^{\Lambda}(y) \} = \min \{ C_{MF}{}^{\Lambda}(p) , C_{MF}{}^{\Lambda}(p) \}$.

Thus $C_{M\,\varsigma (F^{\Lambda})} (pq) \geq \min \{ C_{M\varsigma(F)}{}^{\Lambda}(p) , C_{M\varsigma(F)}{}^{\Lambda}(p) \}$. Similarly, it can be proved that

$C_{N\,\varsigma (F\Lambda)} (pq) \leq \max \{ C_{N\varsigma(F)}{}^{\Lambda}(p) , C_{N\varsigma(F)}{}^{\Lambda}(p) \}$. Also,

$C_{M\,\varsigma (F\Lambda)} (p^{-1}) = \max \{ C_{MF}{}^{\Lambda} (x^{-1}) : \varsigma(x^{-1}) = p^{-1} \} = \max \{ C_{MF}{}^{\Lambda} (x) : \varsigma(x) = p \}= C_{M\,\varsigma (F^{\Lambda})} (p)$. similarly , $C_{M\,\varsigma (F^{\Lambda})} (p^{-1}) = C_{M\,\varsigma (F^{\Lambda})} (p)$.

Theorem 4.3: Let F^{Λ} be two Λ – fermatean fuzzy multi normal subgroup of the group G_1 and $\varsigma : G_1 \rightarrow G_2$ be a bijective homomorphism. Then $\varsigma (F^{\Lambda})$ is an Λ – fermatean fuzzy multi normal subgroup of G_2.

Proof: In view of the given condition , for any two elements $p, q \in G_2$, there exists a unique pair of elements $x ,y \in G$, such that $\varsigma (x) = p$ and $\varsigma (y) = q$.

Consider $\varsigma (F^{\Lambda}) (pq) = (C_{M\,\varsigma (F^{\Lambda})}(pq) , C_{M\,\varsigma (F^{\Lambda})} (pq))$.Then

$C_{M\,\varsigma (F)}{}^{\Lambda} (pq) = \wp (C_{M\,\varsigma (F)} (\varsigma (x) \varsigma (y) , \Lambda) = \wp (C_{M\,\varsigma (F)} (\varsigma (xy) , \Lambda) \geq \wp (C_{MF} (xy) , \Lambda)$ which follows that $= \wp (C_{MF}{}^{\Lambda} (yx) , \Lambda)$ (since $C_{MF}{}^{\Lambda} (xy) = C_{MF}{}^{\Lambda} (yx)$= $\wp \{ C_{M\,\varsigma (F)\ \varsigma (yx)} , \Lambda = \wp \{ C_{M\,\varsigma (F)} (qp), \Lambda \} = C_{M\,\varsigma (F)}{}^{\Lambda} (qp)$, similarly , $C_{M\,(\varsigma (F))}{}^{\Lambda} (pq) = C_{M\,(\varsigma (F))}{}^{\Lambda} (p)$.

The following theorem shows that every Λ – fermatean fuzzy multi inverse homomorphic image of Λ – fermatean fuzzy multi group is always Λ – fermatean fuzzy multi group.

Theorem 4.4: Let $F_2{}^{\Lambda}$ be two Λ – fermatean fuzzy multi group of the group G and $\varsigma : G_1 \rightarrow G_2$ be a group homomorphism. Then $\varsigma^{-1} (F_2{}^{\Lambda})$ is an Λ – fermatean fuzzy multi group of G_1.

Proof: Suppose $F_2{}^{\Lambda}$ be two Λ – fermatean fuzzy multi group of the group G_2 , then there exists a unique pair of elements $x,y \in G_1$ such that

$\varsigma^{-1} (F_2{}^{\Lambda}) (xy) = (C_{M\,\varsigma\text{-}1\,(F_2{}^{\Lambda})} (xy) , C_{N\,\varsigma\text{-}1\,(F_2{}^{\Lambda})} (xy))$. Also

$C_{M\,\varsigma\text{-}1\,(F_2{}^{\Lambda})} (xy) = C_{MF2}{}^{\Lambda} (\varsigma(xy)) = C_{MF2}{}^{\Lambda} (\varsigma(x) \varsigma (y)) \geq \min \{ C_{MF2}{}^{\Lambda} (\varsigma(x)), C_{MF2}{}^{\Lambda} (\varsigma(y) \}$

$= \min \{ C_{M\,\varsigma\text{-}1\,(F_2{}^{\Lambda})} (x) , C_{M\,\varsigma\text{-}1\,(F_2{}^{\Lambda})} (y)\}$

Thus $C_{M\varsigma^{-1}(F2^{\Lambda})}(xy) \geq \min\{C_{M\varsigma^{-1}(F2^{\Lambda})}(x), C_{M\varsigma^{-1}(F2^{\Lambda})}(y)\}$.Similarly, we will show that
$C_{N\varsigma^{-1}(F2^{\Lambda})}(xy) \leq \max\{C_{N\varsigma^{-1}(F2^{\Lambda})}(x), C_{N\varsigma^{-1}(F2^{\Lambda})}(y)\}$. Also $C_{M\varsigma^{-1}(F2^{\Lambda})}(p^{-1}) = C_{MF2}{}^{\Lambda}(\varsigma(p^{-1}))$
$= C_{M(F2)}{}^{\Lambda}(\varsigma(p)^{-1}) = C_{M(F2)}{}^{\Lambda}(\varsigma(p)) = C_{M\varsigma^{-1}(F2^{\Lambda})}(p)$. Similarly,
$C_{N(F2)}{}^{\Lambda}(\varsigma(p)^{-1}) = C_{N(F2)}{}^{\Lambda}(\varsigma(p)) = C_{N\varsigma^{-1}(F2^{\Lambda})}(p)$. consequently $\varsigma^{-1}(F_2{}^{\Lambda})$ is an Λ – fermatean fuzzy multi group of G_1.

The following theorem, we show that every Λ – fermatean fuzzy multi homomorphic inverse image of Λ – fermatean fuzzy multi subgroup of G is an Λ – fermatean fuzzy multi normal subgroup of G.

Theorem 4.5: Let $F_2{}^{\Lambda}$ be two Λ – fermatean fuzzy multi normal subgroup of the group G_2 and $\varsigma : G_1 \rightarrow G_2$ be a group homomorphism. Then $\varsigma^{-1}(F_2{}^{\Lambda})$ is an Λ – fermatean fuzzy multi normal subgroup of G_1.

Proof: Suppose $F_2{}^{\Lambda}$ is an Λ – fermatean fuzzy multi normal group of the group G_2 , then there exists a unique pair of elements $x,y \in G_1$ such that
$\varsigma^{-1}(F_2{}^{\Lambda})(xy) = (C_{M\varsigma^{-1}(F2^{\Lambda})}(xy), C_{N\varsigma^{-1}(F2^{\Lambda})}(xy))$. Also
$C_{M\varsigma^{-1}(F2^{\Lambda})}(xy) = C_{MF2}{}^{\Lambda}(\varsigma(xy)) = C_{MF2}{}^{\Lambda}(\varsigma(x)\varsigma(y)) = C_{MF2}{}^{\Lambda}(\varsigma(y)\varsigma(x)) = C_{MF2}{}^{\Lambda}(\varsigma(yx))$
Similarly, we can prove that $C_{N\varsigma^{-1}(F2^{\Lambda})}(xy) = C_{N\varsigma^{-1}(F2^{\Lambda})}(\varsigma(yx))$. Thus $\varsigma^{-1}(F_2{}^{\Lambda})$ is an Λ – fermatean fuzzy multi normal subgroup of G_1.

CONCLUSION:

In this article, Λ – fermatean fuzzy multi set generalizes the idea of classical fuzzy multi set intending to assess the fuzziness level of uncertainty situation. We have presented cossets, normal subgroups of Λ – fermatean fuzzy multi set and its applications. Also we demonstrated the effectiveness of the image and inverse image of Λ – fermatean fuzzy multi normal subgroup followed by fuzzy multi homomorphism.

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